CHAPTER THREE
CAPACITORS

Introduction:

A capacitor which can also be referred to as a condenser is a device for storing energy. Essentially all capacitors consist of two metal plates, which are separated by an insulator. The insulator is called the dielectric and in certain capacitors, this dielectric can be polystyrene, oil or air.

Charging and discharging a capacitor:

A capacitor is charged when a battery or p.d is connected to it. In order to show that the charged capacitor has stored electricity, a piece of wire is used to connect its terminals and if a spark passes just as the wire makes contact, then electricity has been stored by the capacitor. In order to discharge the capacitor, its plates are joined together.

Capacitance definition and its unit:

- Since Q(charge) is proportional to V(p.d), the ratio $\frac{Q}{V}$is a constant for a capacitor
-The ratio of the charge on either plate, to the potential difference between the plates is called the capacitance, C, of the capacitor.
- Therefore Q = CV =>V = $\frac{Q}{C}$, and C = $\frac{Q}{V}$.

- When V is in volt and Q is in coulomb, then the capacitance C is in farad (F).
- One farad is the capacitance of an extremely large capacitor.
- In practical circuits such as radio receivers, the capacitance of the capacitors used are so small and for this reason, they are therefore expressed in micro farad (UF), where IUF = 10^{-6}F.
- It is also quite usual to express small capacitance, such as those used in record players, in pico farad (PF), where 1pF = 10^{-12}F.

Factors determining capacitance:

These factors are three and these are:

(1) The distance between the plates:

- For a capacitor, $C \propto \frac{1}{d}$, where d = the separation distance between the plates.

-This implies that as the distance between the plates increases, the capacitance decreases, and as this distance decreases, the capacitance increases.

(2) The area of the plates:

-For a capacitor, C is proportional to A, where A = the area of the plate.

-Therefore as the area of the plate increases, the capacitance also increases.

-For the same reason, the capacitance decreases as the area of the plate decreases.

(3) The permittivity, E, of the dielectric between the plates:

-The capacitance is directly proportional to this permittivity.

-Therefore the higher the permittivity, the higher the capacitance and the lower the permittivity, the lower the capacitance.

Some practical capacitors:

- The simplest capacitor consists of two flat parallel plates, with an insulating medium between them.
- A capacitor in which the effective area of the plate can be adjusted, is called a variable capacitor.
- Practical capacitors have a variety of forms, but basically, they are all forms of parallel – plate capacitors.
- The variable air capacitor is used in radio receivers, for tuning to different wavelengths of commercial broadcasting stations.

The capacitance of the parallel plate capacitor:

- Suppose two parallel plates of a capacitor, each have a charge which is numerically equal to Q, then the surface density $= \frac{Q}{A}$, where A = the area of either plate.

- The field intensity between the plates, E, is given by $E = \frac{Q}{\epsilon A}$, where ϵ = the permittivity of the medium concerned.

-Since E is numerically = the potential gradient, then $E = \frac{V}{d}$.

- Now from $E = \frac{Q}{\epsilon A} \Rightarrow \frac{V}{d} = \frac{Q}{\epsilon d}, \Rightarrow \frac{Q}{V} = \frac{\epsilon A}{d}$, and since $\frac{Q}{V} = C$, then $C = \frac{\epsilon A}{d}$, where ϵ is the permittivity of the medium.

- For this reason, a capacitor with parallel plates, having a vacuum or air between them, on the assumption that the permittivity of air is the same as that of vacuum, has a capacitance which is given by $C = \frac{\epsilon_0 A}{d}$, where C = the capacitance in farad (F), A = area of overlap of plates in metres squared, d = distance between the plates in metres.

ϵ_0 = the permittivity of free space = $8.85 \times 10^{-12} fm^{-1}$.

(Q1) State the factors, upon which the capacitance of a parallel plate capacitor depends, and how each factor varies with the capacitance.

Soln:

The capacitance of such a capacitor depends on

(1)The area of the plate.

- Since the capacitance is directly proportional to the area, it increases as the area of the plate increases.

(2)The distance d, between the plates.

Since the capacitance is inversely proportional to d, it decreases as d increases, and vice versa.

(3) The permittivity, ϵ, of the dielectric between the two plates.

- Since the capacitance is directly proportional to the permittivity, it increases as the permittivity of the medium increases and vice versa.

(4)Two horizontal parallel plates, each of area $500cm^2$ are mounted 2mm apart in vacuum. The lower plate is earthed and the upper one is given a positive charge of 0.05uc. Neglecting the edge effect, find

(a) the capacitance between the plates.

(b) the potential of the upper plate.

(c) the electric field strength.

(d) the electric energy stored in the system.

N/B: (1) In changing cm^2 (centimeters squared), into m^2 (metres squared), we multiply by10^{-4}.

-For example $2cm^2 = 2 \times 10^{-4} = 0.0002m^2$.

(2) When changing m^2 (metre squared) into cm^2(centimeter squared), we multiply by 10^4.

- For example $5m^2 = 5 \times 10^4 = 50{,}000cm^2$.

Soln:

$A = 500cm^2 = 500 \times 10^{-4} = 0.05m^2$.

$d = 2mm = \frac{2}{1000} = 0.002m$.

$\epsilon_0 = 8.85 \times 10^{-12}Fm^{-1}$.

$Q = 0.05uc = 0.05 \times 10^{-6}C$.

(a) $C = \frac{\epsilon_0 A}{d} = \frac{8.85 \ X\ 10^{-12} \text{x}\ 0.05}{0.002}$

$= 2.21 \times 10^{-10}F$.

(b)The potential $= V = \frac{Q}{C} = \frac{0.05 \text{ x } 10^{-6}}{2.21 \text{ x } 10^{-10}} = 226v$.

(c)The electric field strength, $E = \frac{V}{d} = \frac{226}{0.002} = 1.13 \times 10^5 vm^{-1}$.

The capacitance of an isolated sphere:

- For this, the capacitance is given by $C = 4\pi\epsilon_0 r$, where r is in metres.
-The ratio of the capacitance with and without the dielectric between the plates, is called the relative permittivity or the dielectric constant of the material used.
- The expression '' without a dielectric'', strictly means the plates of the capacitor, must be placed in vacuum.
- The effect of air on the capacitance of a capacitor is so small that for most purpose,

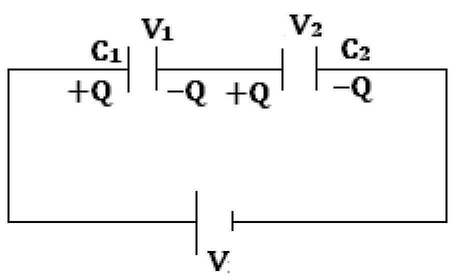
it may be neglected.
- The relative permittivity of a substance is denoted by the letter $\in_r$,

Where $\in_r = \frac{Capacitance\ of\ a\ given\ capacitor, with\ space\ between\ the\ plate\ filled\ with\ dielectic}{Capacitance\ of\ the\ same\ capacitor\ with\ plates\ in\ vacuum}$

- The relative permittivity is the ratio of the permittivity of the substance, to that of free space. i.e. $\in_r = \frac{\in}{\in_0}$, *and it is a pure number which has no dimensions, unlike* $\in$ and $\in_0$.

Capacitors in series:

- When capacitors are arranged in such a manner that the right – hand plate of one, is connected to the left – hand plate of the next, then they are said to have been arranged in series.
- When capacitors are arranged in series, then the charge on each is the same (as the charge put on one of the plates), but the voltage across each is different.



- Consider the figure drawn and assume that a charge, Q, is placed on one of the plates, then the total capacitance of the two capacitors shown in the figure $= \frac{Q}{V}$, where V = the total p.d = $V_1 + V_2$.

- Also $V_1 = \frac{Q}{C_1}$ *and* $V_2 = \frac{Q}{C_2}$.

-The resultant or the equivalent capacitance of these two capacitors is the sum of the reciprocals of their individual capacitances $\Rightarrow \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$.

- In short, to find the resultant or the equivalence capacitance of capacitors in series, we add the reciprocals of their individual capacitances.

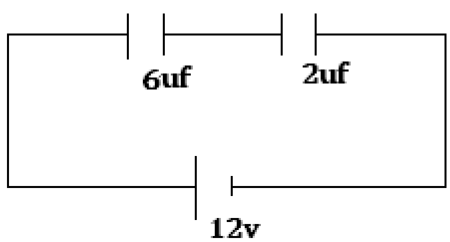
$=> \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$, where C_1, C_2 and C_3 are the individual capacitances, and C = the equivalent capacitance.

(Q3) Two capacitors of charge 6uf and 2uf, respectively are joined in series with a battery of e.m.f 12v. Calculate

(1) the value of the equivalent capacitance in the circuit.

(II) the charge on each capacitor.

Soln:



Let $C_1 = 6uF = 6 \times 10^{-6}F$,

$C_2 = 2uF = 2 \times 10^{-6}F$ and C = the equivalent capacitance of C_1 and C_2.

Since the capacitors are connected in series, then $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$.

In order to make C the subject, we multiply through using CC_1C_2 =>

$CC_1C_2 \times \frac{1}{C} = C\,C_1C_2 \times \frac{1}{C_1} + C\,C_1C_2 \times \frac{1}{C_2}$, $=> C_1C_2 = C\,C_2 + C\,C_1$

$=> C_1C_2 = C\,(C_2 + C_1)$,

Divide through using $C_2 + C_1$

$=> \frac{C_1C_2}{C_2+C_1} = \frac{C(C_2+C_1)}{C_2+C_1}$,

$=> c = \frac{C_1C_2}{C_2+C_1}$,

$=> C = \frac{6 \times 10^{-6} \times 2 \times 10^{-6}}{2 \times 10^{-6} + 6 \times 10^{-6}} = \frac{12 \times 10^{-12}}{8 \times 10^{-6}}$

$= \frac{12}{8} \times 10^{-12} \times 10^{6}$, $=> C = 1.5 \times 10^{-6}C$.

(II) The charge on each capacitor is the same, since the capacitors are connected in series.

From C = $\frac{Q}{V}$ => Q = CV.

=> Charge Q = CV = 1.5 x 10^{-6} x 12 = 18 x 10^{-6}c

N/B: 6 x 10^{-6} + 2 x 10^{-6} = (6 + 2) x 10^{-6} = 8 x 10^{-6}.

-The energy stored in a capacitor = $\frac{0.5Q^2}{C}$, where Q = the charge on (or passing through) the capacitor, and C = the capacitance of the capacitor.